By substitution The method is to re-arrange one of the equations in the form ' $\mathbf{x}=\mathbf{'}^{\prime}$ or ' $\mathbf{y}=\mathbf{'}^{\prime}$ and substitute the value of $\mathbf{x}$ or $\mathbf{y}$ into the second equation.

## Example \#1



$$
\begin{array}{r}
2 x-y=3 \\
5 x-4 y=2
\end{array}
$$

re-arranging (i

$$
\begin{aligned}
y & =2 x-3 \\
5 x-4(2 x-3) & =2 \\
5 x-8 x+12 & =2 \\
-3 x & =2-12 \\
-3 x & =-10 \\
x & =\frac{10}{3} \\
x & =3 \frac{1}{3}
\end{aligned}
$$

substituting for $y$ in (ii
substituting for $x$ in (i)

$$
y=2\left(\frac{10}{3}\right)-3
$$

$$
=\frac{20}{3}-3
$$

$$
=6 \frac{2}{3}-3
$$

$$
=3 \frac{2}{3}
$$

$$
\begin{aligned}
x-2 y & =7 \\
7 x+2 y & =3
\end{aligned}
$$

from ( ${ }^{1}$

$$
\begin{equation*}
x=7+2 y \tag{iii}
\end{equation*}
$$

substituting for $x$ in $(1 i \quad 7(7+2 y)+2 y=3$

$$
49+16 y=3
$$

$$
16 y=3-49
$$

$$
y=\frac{46}{16}=\frac{23}{8}
$$

$$
y=278
$$

substituting for $y$ in (iii

$$
\begin{aligned}
& x=7+2\left(\frac{23}{8}\right) \\
& x=7+\frac{46}{8}=7+5 \frac{3}{4} \\
& x=12 \frac{3}{4}
\end{aligned}
$$

By elimination - Here one equation is altered to make one term in each equation the same(disregarding the +/- sign). These terms are then added or subtracted to eliminate them.

Example \#1

$$
\begin{align*}
2 x-3 y & =5  \tag{i}\\
3 x+y & =2
\end{align*}
$$

multiply (ii by 3, then add (i $\&$ (ii

$$
2 x-3 y=5
$$

$$
9 x+3 y=6
$$

$$
11 x=11
$$

$$
x=1
$$

substituting for $x$ in $(1 \quad 2-3 y=5$

$$
-3 y=5-2
$$

$$
-3 y=3
$$

$$
y=-1
$$

$$
\begin{align*}
5 x-2 y & =1  \tag{1}\\
x-3 y & =3
\end{align*}
$$

multiply (ii by 5 and subtract

$$
\begin{gathered}
5 x-2 y=1 \\
-(5 x-15 y=15)
\end{gathered}
$$

this becomes

$$
\begin{aligned}
& 5 x-2 y=1 \\
& \frac{-5 x+15 y}{}=-15 \\
& \hline 13 y=-14 \\
& y=\frac{-14}{13}, \quad y=-1 \frac{1}{13}
\end{aligned}
$$

substituting for $y$ in equation (ii

$$
\begin{aligned}
x-3\left(\frac{-14}{13}\right) & =3 \\
x+\frac{42}{13} & =3 \\
x & =3-\frac{42}{13} \\
x & =3-3 \frac{3}{13} \\
x & =-\frac{3}{13}
\end{aligned}
$$

$$
\begin{aligned}
& 2 x-5 y=4 \\
& 3 x+6 y=3
\end{aligned}
$$

multiply (i by 3 , multiply (ii by 2 then subtract

$$
6 x-15 y=12
$$

$$
-(6 x+12 y=6)
$$

this becomes:

$$
\begin{aligned}
6 x-15 y & =12 \\
-6 x-12 y & =-6 \\
\hline-27 y & =6 \quad y=-\frac{6}{27} \\
y & =-\frac{2}{9}
\end{aligned}
$$

substituting for $y$ in (i

$$
\begin{aligned}
& 2 x-5\left(-\frac{2}{9}\right)=4 \\
& 2 x+\left(\frac{10}{9}\right)=4 \quad 2 x=4-\left(\frac{10}{9}\right) \\
& 2 x=\left(\frac{36}{9}\right)-\left(\frac{10}{9}\right) \\
& 2 x=\frac{26}{9} \\
& \left.x=\frac{26}{18}, \quad x=\frac{13}{9} \text { (or } 1 \frac{4}{9}\right)
\end{aligned}
$$

Using graphs - For two separate functions, first write tables for $x \& y$. Then draw the graphs. Where the graph lines intersect is the point that satisfies both equations. Simply read off the $x$ and $y$ values at the point.


The reader may wish to verify the result by one of the other methods given above.

Example - by a graphical method find the coodinates of a point that satisfies the equations $x+y=5$ and $y-2 x=-2$

First draw your table, rearranging each equation to make 'y' the subject

| $x$ | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
| $y=5-x$ | 4 | 3 | 2 |
| $y=2 x-2$ | 0 | 2 | 4 |

Then plot the coordinates for each function, drawing straight lines through points.
Where the lines cross gives the solution $(2.3,2.7)$ One decimal place is usually sufficient.


