Simple Equations

Simple Equations are equations with one variable only(usually denoted by 'x').

The 'Golden Rule' - whatever operations are performed, they must operate equally on **both** sides of the equation.

general method for solving equations:

- remove fractions by multiplying both sides of the equation by a common denominator
- expand any brackets
- take 'x'(or variable) terms to the left
- take number terms and unwanted variable terms, to the right
- collect terms on each side
- if the variable(x) on the left is multiplied by a number, divide both sides by that number(the number cancels on the left)
- cancel divided terms where necessary

a practical note:

To move a term from one side of an equation to the other, change its sign.

In effect a term of the same value but opposite in sign is added to each side of the equation. This cancels on one side giving the appearance of the original term having moved across the equation with its sign changed.

$$x + 3 = 0$$
$$x + 3 - 3 = -3$$
$$x = -3$$

In our example -3 is added to each side of the equation. This appears to have the effect of moving the +3 across the equals sign and changing it to -3

Example #1

$$2(x-2) = 3x + 9$$
$$2x - 4 = 3x + 9$$
$$2x - 3x = 9 + 4$$
$$-x = 13$$
$$\underline{x = -13}$$

Example #2

$$3(2x+5)=5(3x-1)$$
$$6x+15=15x-5$$
$$6x-15x=-5-15$$
$$-9x=-20$$
$$9x=20$$
$$x=\frac{20}{9}$$
$$\underline{x=\ 2\tfrac{2}{9}}$$

Example #3

$$\frac{2(x+1)}{3}=\frac{3(2x-2)}{5} \qquad \text{common denominator } 3\times5=15$$
$$\frac{15\times2(x+1)}{3}=\frac{15\times3(2x-2)}{5}$$
$$\frac{30(x+1)}{3}=\frac{45(2x-2)}{5}$$
$$10(x+1)=9(2x-2)$$
$$10x+10=18x-18$$
$$10x-18x=-18-10$$
$$-8x=-28$$
$$x=\frac{28}{8}=\frac{7}{2}$$
$$\underline{x=3\ \tfrac{1}{2}}$$

Substituting values into an equation - Simply write the equation again with the letters replaced by the numbers they represent. Use brackets to avoid arithmetic errors.

Example #1

$$y = 2ax - \frac{3t^2}{2}$$

$a = 2 \quad x = 3 \quad t=2$

$$y = (2\times2\times3) - \frac{3\times2\times2}{2}$$

$$y = 12 - 6$$

$$\underline{y = 6}$$

Example #2

$$y = 4x^2a - \frac{t}{3} + x$$

$a = 2 \quad x = 1 \quad t=3$

$$y = (4\times1\times1\times2) - \frac{3}{3} + 1$$

$$y = 8 - 1 + 1$$

$$\underline{y = 8}$$

Changing the subject of an equation - Simply follow these simple rules:

- move all terms containing the subject to the LHS of the equation either by moving them across the = sign & changing their sign, or flipping the equation over horizontally, so that the LHS is on the right & vice versa.
- factorise the terms containing the subject
- divide both sides of the equation by the contents of any brackets.
- remove any unwanted negative signs on the left by multiplying both sides by -1
- remove fractions by multiplying both sides by the denominator(lower number of any fraction)

Example #1 make '*b*' the subject of the equation

$$p = \frac{xa}{2} - bc^2$$

$$bc^2 = \frac{xa}{2} - p \quad \text{move the } p \text{ and } -bc^2 \text{ across the equals sign}$$

$$2bc^2 = xa - 2p \quad \text{mutliply both sides by 2 to lose the fraction}$$

$$b = \frac{xa}{2c^2} - \frac{2p}{2c^2} \quad \text{divide both sides by } 2c^2 \text{ to get } b \text{ on its own}$$

$$b = \frac{xa}{2c^2} - \frac{p}{c^2} \quad \text{cancel 2}$$

$$b = \frac{xa - 2p}{2c^2} \quad \text{add the fractions}$$

Example #2 make '*d*' the subject of the equation

$$t = c^2 - \frac{ab^2}{3d}$$

$$3dt = 3dc^2 - ab^2 \quad \text{remove fraction, multiply both sides by } 3d$$

$$3dt - 3dc^2 = -ab^2 \quad \text{collect terms in } d \text{ on the LHS}$$

$$d(3t - 3c^2) = -ab^2 \quad \text{factorise, taking the } d \text{ outside the brackets}$$

$$d = \frac{-ab^2}{3(t - c^2)} \quad \text{divide both sides of the equation by } (3t - 3c^2),$$

taking the 3 outside the brackets

Example #3 make 'c' the subject of the equation

$$d - x^2y = \frac{4xd^2}{c}$$

$$cd - cx^2y = 4xd^2$$ remove the fraction by multipling both sides by c

$$c(d - x^2y) = 4xd^2$$ factorise on LHS to extract c

$$c = \frac{4xd^2}{(d - x^2y)}$$ divide both side by $(d - x^2y)$ to leave c by itself on LHS

Creating a formula - Often when creating a formula some basic knowledge is required:

e.g. area of a triangle = $\frac{1}{2}$(base x height), area rectangle = width x length etc.

Example #1 A plant of original length $l_0$cm. grows at a rate of *a* cm. per day. If days are represented by the letter *d* , write an expression for the height *h* of the plant in cm.

$$\text{height } h \text{ of the plant after } d \text{ days} = \text{original height} + \text{growth in } d \text{ days}$$

$$\underline{h = l_0 + ad}$$

Example #2 A box has height *a*, length *b* and width *c* . Write an expression to represent the surface area A, of the box.

$$\text{total surface area (A)} = 2 \text{ end areas} + 4 \text{ side areas}$$

$$\underline{\text{A} = 2ac + 2ab + 2bc}$$

Example #3 A square box of side L contains a sphere, where the radius of the sphere is equal to half the side of the box. Write an expression for the volume in the box not taken up by the sphere.

required volume V = volume of the box - volume of the sphere

$$= L^3 - \frac{4}{3}\pi r^3 \qquad \text{but } r = \frac{L}{2}$$

$$= L^3 - \frac{4}{3}\pi\left(\frac{L}{2}\right)^3$$

$$= L^3 - \frac{4}{3}\pi\frac{L^3}{8} \qquad \text{cancelling 4,8}$$

$$= L^3 - \frac{1}{3}\pi\frac{L^3}{2} = L^3 - \pi\frac{L^3}{6}$$

$$\underline{V = L^3\left(1 - \frac{\pi}{6}\right)}$$