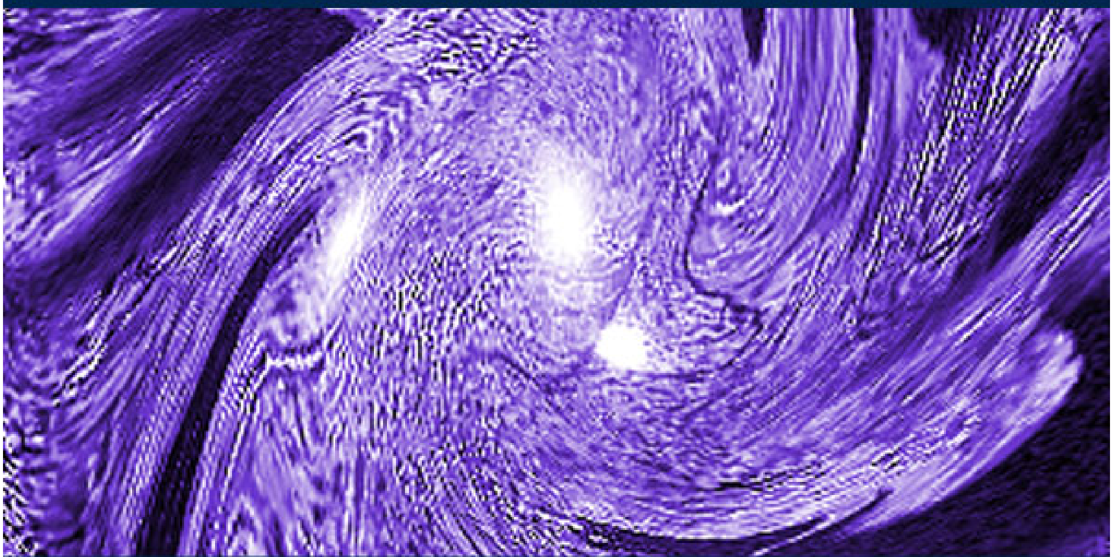
GCSE MATHS TUTOR

Revision Guide



PART TWO

ALGEBRA

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Contents

Basics

Short-hand

$4a$	$4\times a$
$3abc$	$3\times a\times b\times c$
ab^2	$a\times b\times b$
$(cd)^2$	$c\times d\times c\times d$

Rules of Sign

$$(+)\text{x}(+) = +$$

....

$$(+)\text{x}(-) = -$$

....

$$(-)\text{x}(+) = -$$

....

$$(-)\text{x}(-) = +$$

$$(+3a) \times (+2b) = +6ab$$

$$(+2a) \times (-4b) = -8ab$$

$$(-6a) \times (+3b) = -18ab$$

$$(-5a) \times (-6b) = +30ab$$

$$(+2a) \times (+b) = +2ab$$

$$(+a) \times (-3b) = -3ab$$

$$(-7a) \times (-5b) = +35ab$$

$$\frac{+}{+} = +$$

....

$$\frac{+}{-} = -$$

....

$$\frac{-}{+} = -$$

....

$$\frac{-}{-} = +$$

$$\frac{+5a}{+6b} = \frac{5a}{6b}$$

$$\frac{-3a}{+7b} = -\frac{3a}{7b}$$

$$\frac{+2a}{-3b} = -\frac{2a}{3b}$$

$$\frac{-a}{-4b} = +\frac{a}{4b} = \frac{a}{4b}$$

Terms - A term is a collection of letters and/or numbers multiplied together, with a '+' or '-' sign infront of it.

Terms are referred to as 'the term in ...' or 'the ... term'.

For example, '2y' is the term in 'y' or the '2y' term.

examples of terms... $+2x^2$ $-3xy$ $-5x$ $+6y^3$ -3

Simplifying/Collecting Terms - This involves grouping terms together and adding them.

Example #1

$+6x - 2 + x + 7 - 3x - 4$

$+6x + x - 3x \qquad -2 + 7 - 4$

$+4x \qquad +1$

$+4x + 1$

Example #2

$3x^2 - 2 + 2y - 1 + xy - 3y - x^2$

$3x^2 - x^2 \qquad + xy \qquad + 2y - 3y \qquad - 2 - 1$

$2x^2 \qquad + xy \qquad - y \qquad - 3$

$2x^2 + xy - y - 3$

Example #3

$5x^2 - y + 3 - y^2 + 3y - 2x^2 + 2y$

$5x^2 - 2x^2 \qquad - y + 3y + 2y \qquad + 3 \qquad - y^2$

$3x^2 \qquad + 4y \qquad + 3 \qquad - y^2$

$3x^2 + 3 + 4y - y^2$

Brackets

Multiplying Out(expanding) - a pair of brackets with a single term infront

The term outside the brackets multiplies each of the terms in turn inside the brackets.

example:

$$\begin{aligned} & x(a+b+c) \\ = & xa+xb+xc \end{aligned}$$

further examples:

$$\begin{aligned} & 3p(2x-5y) \\ = & 6px-15py \end{aligned}$$

$$\begin{aligned} & 2xy(p+2q) \\ = & 2xyp+4xyq \end{aligned}$$

$$\begin{aligned} & 3x^2(y+4z^2) \\ = & 3x^2y+12x^2z^2 \end{aligned}$$

$$\begin{aligned} & -2x(3p^2-2q^2) \\ = & -6xp^2+4xq^2 \end{aligned}$$

$$\begin{aligned} & 3q^3(5r^3-2p^2) \\ = & 15q^3r^3-6q^3p^2 \end{aligned}$$

$$\begin{aligned} & 2r^2(6x-y) \\ = & 12r^2x-2r^2y \end{aligned}$$

Multiplying Out(expanding) - two pairs of brackets

Think of the two terms in the first bracket as separate single terms infront of a pair of brackets.

example:

$$(3a-2b)(a+b)$$

Multiply the **contents** of the 2nd bracket by the **1st term** in the 1st bracket.

$$(\underline{3a}-2b)(\underline{a+b})$$
$$3a(a+b)$$
$$=3a^2+3ab$$

Multiply the **contents** of the 2nd bracket by the **2nd term** in the 1st bracket.

$$(3a\underline{-2b})(\underline{a+b})$$
$$-2b(a+b)$$
$$=-2ab-2b^2$$

Add the two results together.

$$\begin{array}{l} 3a^2+3ab \\ \underline{\qquad -2ab-2b^2} \\ 3a^2+ab-2b^2 \end{array}$$

Example #1

$$
\begin{array}{l}
(7x-5)(2x-3) \\
14x^2-21x \\
\underline{\qquad -10x+15} \\
14x^2-31x+15
\end{array}
$$

Example #2

$$
\begin{array}{l}
(2x-2)(5x+3) \\
10x^2+6x \\
\underline{\qquad -10x-6} \\
10x^2-4x-6
\end{array}
$$

Example #3

$$
\begin{array}{l}
(2x+9)(3x-11) \\
6x^2-22x \\
\underline{\qquad +27x-99} \\
6x^2+5x-99
\end{array}
$$

Squared Brackets

$$
\begin{aligned}
(x+y)^2 &= (x+y)(x+y) \\
&= x^2 + xy \\
&\quad\;\; \underline{+ xy + y^2} \\
&\quad\; x^2 + 2xy + y^2
\end{aligned}
$$

note: a common mistake

$$(x+y)^2 \neq x^2 + y^2$$

Difference of Two Squares

Brackets - Simple Factorising - This involves taking out a common term from each expression and placing it infront of the brackets.

examples:

$$3x^2 - 9x \qquad\qquad 4x^3 - 6x^2$$

$$\underline{3x(x-3)} \qquad\qquad \underline{2x^2(2x-3)}$$

$$5x^2y - 10xy^2 \qquad\qquad 8x - 12xy$$

$$\underline{5xy(x-2y)} \qquad\qquad \underline{4x(2-3y)}$$

$$3xy^3 - 15x^2y \qquad\qquad 7x^2y^2 - 21xy$$

$$\underline{3xy(y^2-5x)} \qquad\qquad \underline{7xy(xy-3)}$$

Factorising Quadratic Expressions

This is best illustrated with an example:

$$x^2 - 7x + 12$$

You must first ask yourself which two factors when multiplied will give **12** ?

The factors of 12 are : ....**1 x 12**, ...**2 x 6**, ...**3 x 4**

Now which numbers in a group added or subtracted will give **7** ?

1 : 12 gives 13, 11 ....2 : 6 gives 8, 4 ....**3 : 4** gives **7**, 1

so

$$x^2 - 7x + 12 = (x \pm 3)(x \pm 4)$$

which of the '+' & '-' terms makes **+12**? ....and when added gives **-7**?

these are the choices: (+3)(+4), (-3)(+4), (+3)(-4) or **(-3)(-4)**

clearly, **(-3)(-4)** are the two factors we want

therefore

$$\underline{x^2 - 7x + 12 = (x - 3)(x - 4)}$$

Example #1

$$x^2 - x - 20$$

$$(x \pm 5)(x \pm 4)$$

$$\underline{(x - 5)(x + 4)}$$

Example #2

$$x^2 + x - 42$$

$$(x \pm 7)(x \pm 6)$$

$$\underline{(x + 7)(x - 6)}$$

Example #3

$$x^2 - 13x + 30$$

$$(x \pm 10)(x \pm 3)$$

$$\underline{(x - 10)(x - 3)}$$

Indices

The Laws of Indices have been examined already with respect to 'number' under the heading 'powers & roots'.

However, in this section indices will be looked at in more depth, this time examples will use algebraic symbols.

The Laws of Indices

$$p^m \times p^n = p^{m+n}$$

$$\frac{p^m}{p^n} = p^{m-n}$$

$$(p^m)^n = p^{m \times n} = p^{mn}$$

$$\sqrt[n]{p} = p^{\frac{1}{n}}$$

$$\frac{1}{p^m} = p^{-m} \qquad p^n = \frac{1}{p^{-n}}$$

$$p^0 = 1$$

Indices - Multiplication

remembering that:

$$p^m \times p^n = p^{m+n}$$

Examples

$a^2 \times a^5 = a^7$ $\quad a^3b^2 \times a^4b^7 = a^7b^9$ $\quad a^{-2}b^3 \times a^5b^{-4} = a^3b^{-1}$

$2a^3b^2c^{-3} \times 5a^3b^{-2}c^2 = 10a^6b^0c^{-1} = 10a^6c^{-1}$ $\quad (b^0 = 1)$

$5a^2b^{-7}c^{-2} \times 6a^{-2}b^5c^3 = 30a^0b^{-2}c^1 = 30b^{-2}c$ $\quad (a^0 = 1,\ \ c^1 = c)$

Indices - Division

remembering that:

$$\frac{p^m}{p^n} = p^{m-n}$$

Examples:

$\frac{a^5}{a^6} = a^{-1}$ $\quad \frac{a^3b^5}{a^2b^7} = a^1b^{-2} = ab^{-2}$

$\frac{a^6b^3}{a^7b^{-5}} = a^{-1}b^{3-(-5)} = a^{-1}b^8$ $\quad \frac{12a^3b^2}{3a^{-4}b^4} = 4a^{3-(-4)}b^{-2} = 4a^7b^{-2}$

$\frac{8a^4b^2c^{-7}}{2a^3b^{-4}c^{-5}} = 4a^1b^{2-(-4)}c^{-7-(-5)} = 4ab^6c^{-2}$

Indices - Powers

remembering that:

$$(p^m)^n = p^{m\times n} = p^{mn}$$

Examples:

$$(a^3b^5)^3 = a^9b^{15} \qquad (a^4b^2)^{-5} = a^{-20}b^{-10} \qquad (a^{-2}b^4)^{-3} = a^6b^{-12}$$

$$(3a^2b)^3 = 27a^6b^3 \qquad (2a^3b^4)^3 = 8a^9b^{12} \qquad (4ab^3)^2 = 16a^2b^6$$

$$4(2ab^2)^3 = 4(8a^3b^6) = 32a^3b^6, \qquad 3(4a^4b^3)^2 = 3(16a^8b^6) = 48a^8b^6$$

Indices - Roots and Reciprocals

remembering that:

$$\sqrt[n]{p} = p^{\frac{1}{n}}$$

and

$$\frac{1}{p^m} = p^{-m} \qquad p^n = \frac{1}{p^{-n}}$$

Examples:

$$\frac{a^3}{b^4} = a^3b^{-4} \qquad \frac{a^2}{b^{-3}c^2} = a^2b^3c^{-2} \qquad \frac{ab^2c^3}{b^{-5}c^2} = ab^7c$$

$$\sqrt{\frac{a^2}{b^6}} = \left(a^2b^{-6}\right)^{1/2} = a^{2/2}b^{-6/2} = ab^{-3} \qquad \sqrt[3]{\frac{b^4}{a^{-7}}} = \left(a^7b^4\right)^{1/3} = a^{7/3}b^{4/3}$$

$$\frac{a^2b^{1/3}}{a^{1/2}b^3} = a^2\ a^{-1/2}\ b^{-3}\ b^{1/3} = a^{4/2}\ a^{-1/2}\ b^{-9/3}\ b^{1/3} = a^{3/2}\ b^{-8/3}$$

Algebraic Fractions

Algebraic Fractions - Addition

This is just like number fraction addition, but with symbols.

To add two fractions you must first find their common denominator. Then convert each to the new denominator and add the new numerators.

The common denominator is found by multiplying the two numerators together.

$$\frac{2x}{3y}+\frac{5y}{4x}$$

$$\frac{}{12xy}$$

In this case, multiply the **3y** by the **4x**. This gives**12xy**.

Now convert each factor to a factor of 12xy by dividing the denominator of each into 12xy and multiplying the result by each numerator(2x,5y)

$$\frac{2x}{3y}+\frac{5y}{4x}$$

$$\frac{4x(2x)+3y(5y)}{12xy}$$

$$=\frac{8x^2+15y^2}{12xy}$$

Example #1

$$\frac{4x}{3y}+\frac{2ab}{5c}$$

$$\frac{5c(4x)+3y(2ab)}{15cy}$$

$$=\frac{20cx+6aby}{15cy}$$

Example #2

$$\frac{x^2}{2c}+\frac{3a^3b}{7xy}$$

$$\frac{7xy(x^2)+2c(3a^3b)}{14cxy}$$

$$=\frac{7x^3y+6a^3bc}{14cxy}$$

Algebraic Fractions - Subtraction - The method here is similar to addition, except the numerators(top terms) in the new fraction are subtracted.

Example #1

$$\frac{a^2}{5x} - \frac{2b^3y}{3ac}$$
$$\frac{3ac(a^2) - 5x(2b^3y)}{15acx}$$

$$= \frac{3a^3c - 10b^3xy}{15acx}$$

Example #2

$$\frac{p^3}{3r^2} - \frac{5q^4r}{3pr}$$
$$\frac{p(p^3) - r(5q^4r)}{3pr^2}$$

$$= \frac{p^4 - 5q^4r^2}{3pr^2}$$

Algebraic Fractions - Multiplication - Simply multiply across the denominators and the numerators, keeping them separate. Cancel any terms where possible.

Example #1

$$\frac{3cp^2}{4r^3}\times\frac{6b^4c^2}{5ar^2}=\frac{3cp^2\times 6b^4c^2}{4r^3\times 5ar^2}=\frac{18b^4c^3p^2}{20ar^5}=\frac{9b^4c^3p^2}{10ar^5}$$

Example #2

$$\frac{4x^3y^2}{5t^3z}\times\frac{3y^2z^5}{5t^2x}=\frac{4x^3y^2\times 3y^2z^5}{5t^3z\times 5t^2x}=\frac{12x^3y^4z^5}{25t^5xz}=\frac{12x^2y^4z^4}{25t^5}$$

Algebraic Fractions - Division - Simply invert the term you divide by(the 2nd term), and procede as for multiplication.

Example #1

$$\frac{2b^5x^2z}{5t^2y^3}\div\frac{4b^2z^5}{5xy^2}=\frac{2b^5x^2z}{5t^2y^3}\times\frac{5xy^2}{4b^2z^5}=\frac{2b^5x^2z\times 5xy^2}{5t^2y^3\times 4b^2z^5}=\frac{10b^5x^3y^2z}{20b^2t^2y^3z^5}$$
$$=\frac{10b^3x^3}{20t^2yz^4}$$

Example #2

$$\frac{5b^2x^3z}{6t^5y^4}\div\frac{7b^2z^2}{3x^3y^3}=\quad\frac{5b^2x^3z}{6t^5y^4}\times\frac{3x^3y^3}{7b^2z^2}=\frac{5b^2x^3z\times 3x^3y^3}{6t^5y^4\times 7b^2z^2}$$
$$=\frac{15b^2x^6y^3z}{42b^2t^5y^4z^2}=\frac{5x^6}{14t^5yz}$$

Simple Equations

Simple Equations are equations with one variable only(usually denoted by 'x').

The 'Golden Rule' - whatever operations are performed, they must operate equally on **both** sides of the equation.

general method for solving equations:

- remove fractions by multiplying both sides of the equation by a common denominator
- expand any brackets
- take 'x'(or variable) terms to the left
- take number terms and unwanted variable terms, to the right
- collect terms on each side
- if the variable(x) on the left is multiplied by a number, divide both sides by that number(the number cancels on the left)
- cancel divided terms where necessary

a practical note:

To move a term from one side of an equation to the other, change its sign.

In effect a term of the same value but opposite in sign is added to each side of the equation. This cancels on one side giving the appearance of the original term having moved across the equation with its sign changed.

$$x + 3 = 0$$
$$x + 3 - 3 = -3$$
$$x = -3$$

In our example -3 is added to each side of the equation. This appears to have the effect of moving the +3 across the equals sign and changing it to -3

Example #1

$$2(x-2) = 3x + 9$$
$$2x - 4 = 3x + 9$$
$$2x - 3x = 9 + 4$$
$$-x = 13$$
$$\underline{x = -13}$$

Example #2

$$3(2x+5) = 5(3x-1)$$
$$6x+15 = 15x-5$$
$$6x-15x = -5-15$$
$$-9x = -20$$
$$9x = 20$$
$$x = \frac{20}{9}$$
$$\underline{x = \ 2\tfrac{2}{9}}$$

Example #3

$$\frac{2(x+1)}{3} = \frac{3(2x-2)}{5} \qquad \text{common denominator } 3\times 5 = 15$$
$$\frac{15\times 2(x+1)}{3} = \frac{15\times 3(2x-2)}{5}$$
$$\frac{30(x+1)}{3} = \frac{45(2x-2)}{5}$$
$$10(x+1) = 9(2x-2)$$

$$10x+10 = 18x-18$$

$$10x-18x = -18-10$$

$$-8x = -28$$

$$x = \frac{28}{8} \ = \frac{7}{2}$$

$$\underline{x \ = 3} \ \frac{1}{2}$$

Substituting values into an equation - Simply write the equation again with the letters replaced by the numbers they represent. Use brackets to avoid arithmetic errors.

Example #1

$$y = 2ax - \frac{3t^2}{2}$$

$a = 2 \quad x = 3 \quad t=2$

$$y = (2\times2\times3) - \frac{3\times2\times2}{2}$$

$$y = 12 - 6$$

$$\underline{y = 6}$$

Example #2

$$y = 4x^2a - \frac{t}{3} + x$$

$a = 2 \quad x = 1 \quad t=3$

$$y = (4\times1\times1\times2) - \frac{3}{3} + 1$$

$$y = 8 - 1 + 1$$

$$\underline{y = 8}$$

Changing the subject of an equation - Simply follow these simple rules:

- move all terms containing the subject to the LHS of the equation either by moving them across the = sign & changing their sign, or flipping the equation over horizontally, so that the LHS is on the right & vice versa.
- factorise the terms containing the subject
- divide both sides of the equation by the contents of any brackets.
- remove any unwanted negative signs on the left by multiplying both sides by -1
- remove fractions by multiplying both sides by the denominator(lower number of any fraction)

Example #1 make '*b*' the subject of the equation

$$p = \frac{xa}{2} - bc^2$$

$$bc^2 = \frac{xa}{2} - p \qquad \text{move the } p \text{ and } -bc^2 \text{ across the equals sign}$$

$$2bc^2 = xa - 2p \qquad \text{mutliply both sides by 2 to lose the fraction}$$

$$b = \frac{xa}{2c^2} - \frac{2p}{2c^2} \qquad \text{divide both sides by } 2c^2 \text{ to get } b \text{ on its own}$$

$$b = \frac{xa}{2c^2} - \frac{p}{c^2} \qquad \text{cancel 2}$$

$$b = \frac{xa - 2p}{2c^2} \qquad \text{add the fractions}$$

Example #2 make '*d*' the subject of the equation

$$t = c^2 - \frac{ab^2}{3d}$$

$$3dt = 3dc^2 - ab^2 \qquad \text{remove fraction, multiply both sides by } 3d$$

$$3dt - 3dc^2 = -ab^2 \qquad \text{collect terms in } d \text{ on the LHS}$$

$$d(3t - 3c^2) = -ab^2 \qquad \text{factorise, taking the } d \text{ outside the brackets}$$

$$d = \frac{-ab^2}{3(t - c^2)} \qquad \text{divide both sides of the equation by } (3t - 3c^2),$$

taking the 3 outside the brackets

<u>Example #3</u> make 'c' the subject of the equation

$$d - x^2y = \frac{4xd^2}{c}$$

$$cd - cx^2y = 4xd^2 \qquad \text{remove the fraction by multipling both sides by } c$$

$$c(d - x^2y) = 4xd^2 \qquad \text{factorise on LHS to extract } c$$

$$c = \frac{4xd^2}{(d - x^2y)} \quad \text{divide both side by } (d - x^2y) \text{ to leave } c$$

by itself on LHS

<u>Creating a formula</u> - Often when creating a formula some basic knowledge is required:

e.g. area of a triangle = $\frac{1}{2}(\text{base x height})$, area rectangle = width x length etc.

<u>Example #1</u> A plant of original length $l_0$cm. grows at a rate of *a* cm. per day. If days are represented by the letter *d* , write an expression for the height *h* of the plant in cm.

$$\text{height } h \text{ of the plant after } d \text{ days} = \text{original height} + \text{growth in } d \text{ days}$$

$$\underline{h = l_0 + ad}$$

<u>Example #2</u> A box has height *a*, length *b* and width *c* . Write an expression to represent the surface area A, of the box.

$$\text{total surface area (A)} = 2 \text{ end areas} + 4 \text{ side areas}$$

$$\underline{\mathrm{A} = 2ac + 2ab + 2bc}$$

<u>Example #3</u> A square box of side L contains a sphere, where the radius of the sphere is equal to half the side of the box. Write an expression for the volume in the box not taken up by the sphere.

$$\text{required volume V} = \text{volume of the box - volume of the sphere}$$

$$= L^3 - \frac{4}{3}\pi r^3 \qquad \text{but } r = \frac{L}{2}$$

$$= L^3 - \frac{4}{3}\pi\left(\frac{L}{2}\right)^3$$

$$= L^3 - \frac{4}{3}\pi\frac{L^3}{8} \qquad \text{cancelling } 4,8$$

$$= L^3 - \frac{1}{3}\pi\frac{L^3}{2} = L^3 - \pi\frac{L^3}{6}$$

$$\underline{V = L^3\left(1 - \frac{\pi}{6}\right)}$$

Simultaneous Equations

By substitution The method is to re-arrange one of the equations in the form '***x*=**' or '***y*=**' and substitute the value of ***x*** or ***y*** into the second equation.

Example #1

$$3x - 5y = 2 \qquad \text{(i}$$

$$x + 2y = 3 \qquad \text{(ii}$$

re-arranging(ii $\qquad x = 3 - 2y \qquad$ (iii

substituting into (i for x $\qquad 3(3 - 2y) - 5y = 2$

$$9 - 6y - 5y = 2$$

$$9 - 2 = 6y + 5y$$

$$6y + 5y = 9 - 2$$

$$11y = 7$$

$$\underline{y = \frac{7}{11}}$$

substituting for y in (iii $\qquad x = 3 - 2\tfrac{7}{11}$

$$x = 3 - \tfrac{14}{11}$$

$$x = 3 - 1\tfrac{3}{11}$$

$$\underline{x = 1\tfrac{8}{11}}$$

Example #2

$$2x - y = 3 \qquad \text{(i}$$
$$5x - 4y = 2 \qquad \text{(ii}$$

re-arranging (i $\qquad y = 2x - 3$

substituting for y in (ii $\qquad 5x - 4(2x - 3) = 2$

$$5x - 8x + 12 = 2$$
$$-3x = 2 - 12$$
$$-3x = -10$$
$$x = \frac{10}{3}$$
$$\underline{x = 3\tfrac{1}{3}}$$

substituting for x in (i $\qquad y = 2\left(\frac{10}{3}\right) - 3$

$$= \frac{20}{3} - 3$$
$$= 6\tfrac{2}{3} - 3$$
$$\underline{= 3\tfrac{2}{3}}$$

Example #3

$$x - 2y = 7 \qquad \text{(i}$$
$$7x + 2y = 3 \qquad \text{(ii}$$

from (i $\qquad x = 7 + 2y \qquad$ (iii

substituting for x in (ii $\quad 7(7 + 2y) + 2y = 3$

$$49 + 16y = 3$$
$$16y = 3 - 49$$
$$y = \frac{46}{16} = \frac{23}{8}$$
$$\underline{y = 2\,{}^{7}\!/_{8}}$$

substituting for y in (iii $\qquad x = 7 + 2\left(\frac{23}{8}\right)$

$$x = 7 + \frac{46}{8} = 7 + 5\tfrac{3}{4}$$
$$\underline{x = 12\tfrac{3}{4}}$$

By elimination - Here one equation is altered to make one term in each equation the same(disregarding the +/- sign). These terms are then added or subtracted to eliminate them.

Example #1

$$2x - 3y = 5 \qquad (i$$

$$3x + y = 2 \qquad (ii$$

multiply (ii by 3, then add (i & (ii

$$2x - 3y = 5$$

$$\underline{9x + 3y = 6}$$

$$11x \quad = 11$$

$$\underline{x = 1}$$

substituting for x in (i $\quad 2 - 3y = 5$

$$-3y = 5 - 2$$

$$-3y = 3$$

$$\underline{y = -1}$$

Example #2

$$5x - 2y = 1 \qquad \text{(i}$$
$$x - 3y = 3 \qquad \text{(ii}$$

multiply (ii by 5 and subtract

$$5x - 2y = 1$$
$$\underline{-(5x - 15y = 15)}$$

this becomes :

$$5x - 2y = 1$$
$$\underline{-5x + 15y = -15}$$
$$13y = -14$$

$$y = \frac{-14}{13}, \qquad \underline{y = -1\tfrac{1}{13}}$$

substituting for y in equation (ii

$$x - 3\left(\frac{-14}{13}\right) = 3$$
$$x + \frac{42}{13} = 3$$
$$x = 3 - \frac{42}{13}$$
$$x = 3 - 3\tfrac{3}{13}$$
$$\underline{x = -\tfrac{3}{13}}$$

Example #3

$$2x - 5y = 4 \qquad \text{(i}$$
$$3x + 6y = 3 \qquad \text{(ii}$$

multiply (i by 3 , multiply (ii by 2 then subtract

$$6x - 15y = 12$$
$$\underline{-\ (6x + 12y = 6)}$$

this becomes:

$$6x - 15y = 12$$
$$\underline{-\ 6x - 12y = -6}$$

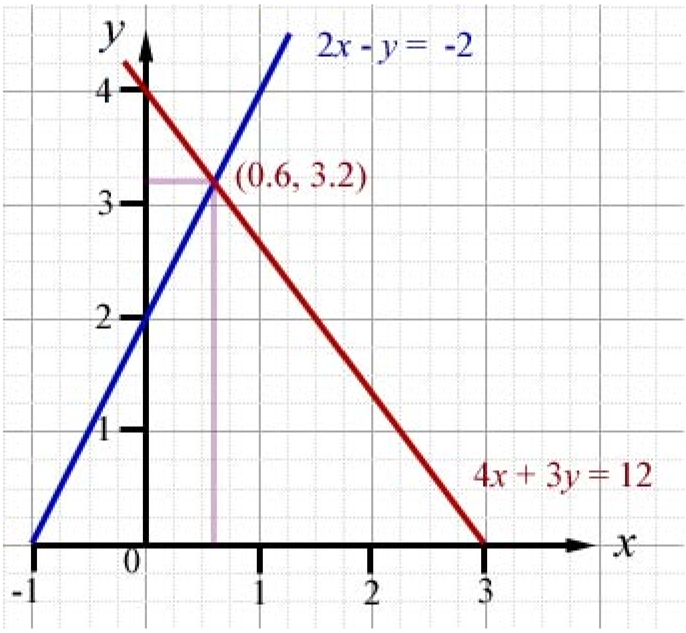
$$-27y = 6 \qquad y = -\frac{6}{27}$$

$$\underline{y = -\frac{2}{9}}$$

substituting for y in (i

$$2x - 5\left(-\frac{2}{9}\right) = 4$$
$$2x + \left(\frac{10}{9}\right) = 4 \qquad 2x = 4 - \left(\frac{10}{9}\right)$$
$$2x = \left(\frac{36}{9}\right) - \left(\frac{10}{9}\right)$$
$$2x = \frac{26}{9}$$
$$x = \frac{26}{18}, \qquad \underline{x = \frac{13}{9}} \quad (\text{or } 1\tfrac{4}{9})$$

<u>Using graphs</u> - For two separate functions, first write tables for x & y. Then draw the graphs. Where the graph lines intersect is the point that satisfies both equations. Simply read off the x and y values at the point.



The reader may wish to verify the result by one of the other methods given above.

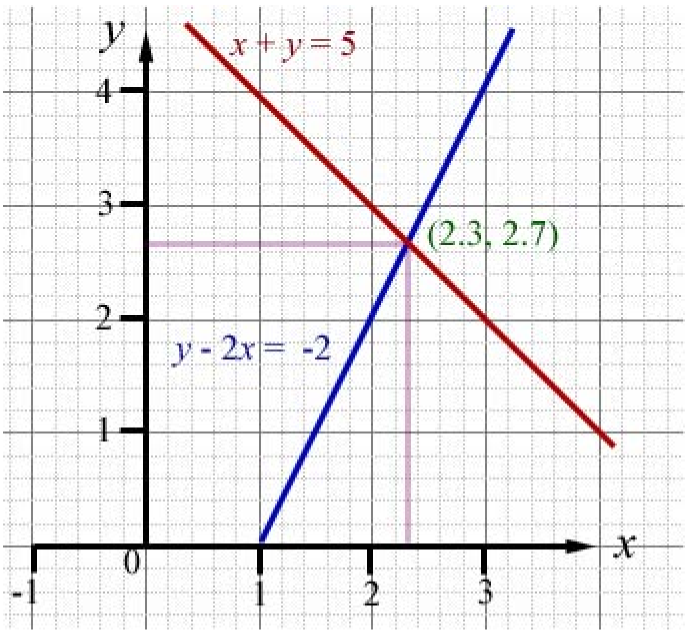
<u>Example</u> - by a graphical method find the coordinates of a point that satisfies the equations $x+y=5$ and $y-2x=-2$

First draw your table, rearranging each equation to make 'y' the subject

x	1	2	3
y = 5-x	4	3	2
y = 2x-2	0	2	4

Then plot the coordinates for each function, drawing straight lines through points.

Where the lines cross gives the solution (2.3,2.7) One decimal place is usually sufficient.



Quadratic Equations

Solution by factorising - This is best understood with an example.

solve: $x^2 - 7x + 12 = 0$

You must first ask yourself which two factors when multiplied will give **12 ?**

The factor pairs of **12** are : 1 x 12, 2 x 6 and 3 x 4

You must decide which of these factor pairs added or subtracted will give **7** ?

1 : 12 ...gives 13, 11

2 : 6 .....gives 8, 4

3 : 4 .....gives 7, 1

so $x^2 - 7x + 12 = (x \pm 3)(x \pm 4)$

Which combination when multiplied makes +12 and which when added gives -7?

these are the choices:

(+3)(+4),

(-3)(+4),

(+3)(-4)

(-3)(-4)

Clearly, (-3)(-4) are the two factors we want.

therefore

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

Now to solve the equation

$$x^2 - 7x + 12 = 0.$$

factorising, as above

$$(x-3)(x-4) = 0$$

either

$$(x-3) = 0$$

or

$$(x-4) = 0$$

for the equation to be true.

So the roots of the equation are:

$$x = 3, \qquad x = 4$$

Completing the square

This can be fraught with difficulty, especially if you only half understand what you are doing.

The method is to move the last term of the quadratic over to the right hand side of the equation and to add a number to both sides so that the left hand side can be factorised as the square of two terms.
e.g.

$$
\begin{aligned}
x^2 - 4x - 5 &= 0 \\
x^2 - 4x &= 5 \\
x^2 - 4x + 4 &= 5 + 4 \\
x^2 - 4x + 4 &= 9 \\
(x-2)(x-2) &= 9 \\
(x-2) &= \pm 3 \\
x - 2 = +3\ ,\ \ x &= +3 + 2\ ,\ \underline{x = 5} \\
x - 2 = -3\ ,\ \ x &= -3 + 2\ ,\ \underline{x = -1}
\end{aligned}
$$

However, there is a much neater way of solving this type of problem, and that is by remembering to put the equation in the following form:

$$
\begin{aligned}
ax^2 + bx + c &= 0 \\
x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 \\
&= \left(x + \frac{b}{2a}\right)^2 + D
\end{aligned}
$$

using the previous example,

$$x^2 - 4x - 5 = 0 \qquad a = 1, \qquad b = -4, \qquad c = -5$$

$$x^2 - 4x - 5 = \left(x + \frac{-4}{2\times1}\right)^2 + D$$

$$x^2 - 4x - 5 = \left(x + (-2)\right)^2 + D$$

$$x^2 - 4x - 5 = \left(x - 2\right)^2 + D$$

$$x^2 - 4x - 5 = x^2 - 4x + 4 + D$$

$$-5 = 4 + D \text{ , } \underline{D = -9}$$

$$\left(x - 2\right)^2 + D = 0$$

$$\left(x - 2\right)^2 - 9 = 0$$

$$\left(x - 2\right)^2 = 9$$

$$(x - 2)(x - 2) = 9$$

$$(x - 2) = \pm 3$$

$$x - 2 = +3 \text{ , } x = +3 + 2 \text{ , } \underline{x = 5}$$

$$x - 2 = -3 \text{ , } x = -3 + 2 \text{ , } \underline{x = -1}$$

Using the Formula - the two solutions of quadratic equations in the form

$$ax^2 + bx + c = 0$$

are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example find the two values of x that satisfy the following quadratic equation:

$$2x^2 + 5x - 4 = 0$$

$$a = 2, \quad b = 5, \quad c = -4$$

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-5 \pm \sqrt{5^2 - 4(2)(-4)}}{2 \times 2} \\
&= \frac{-5 \pm \sqrt{25 + 32}}{4} \\
&= \frac{-5 \pm \sqrt{57}}{4} \\
&= -\frac{5}{4} \pm \frac{\sqrt{57}}{4} \\
&= -1.25 \pm 1.89
\end{aligned}$$

$$x = -1.25 + 1.89, \quad \underline{x = 0.64}$$

$$x = -1.25 - 1.89, \quad \underline{x = -3.11}$$

Trial & Improvement

Method - This is a way of finding **one root** of a quadratic(or cubic)equation, essentially by trial and error, but in a more ordered way.

In questions you are given a quadratic expression and told what numerical value it is equal to.

You are sometimes given that the value of 'x' lies between two numbers and asked that your answer be rounded to a particular number of decimal places. You may wish to revise the topic 'approximation' before proceeding.

Example - By using a 'trial & improvement' method find the positive value of x that satisfies the equation below. X has a value between 1 and 2. Give your answer correct to **one** decimal place.

$$x^2 - 6x - 10 = 0$$

Move the number part of the equation to the right, leaving terms in 'x' on the left.

$$x^2 + 6x = 10$$
$$2^2 + 6(2) = 4 + 12 = 16$$
$$1^2 + 6(1) = 1 + 6 = 7$$

Remember we want the expression to equal **10**. The value of the expression in the first instance comes to **16** , while in the second instance the value is **7**.

Since (16 - 10 = 4) and (10 - 7 = 3) the value of x (to give **10**) must be a number closer to x=1 than x=2.

So we try x=1.4

(1.5 is the average between 2 & 1, so we use 1.5 - 0.1 to give a bias towards the 1).

$$(1.4)^2 + 6(1.4) = 1.96 + 8.4 = 10.36$$

The value of x must be less than 1.4, since 1.4 gives 10.36(over 10).

Let's try x=1.35

$$(1.35)^2 + 6(1.35) = 1.823 + 8.1 = 9.923$$

The value of x must be greater than 1.35, since 1.35 gives 9.923(less than 10).

Let's try x=1.36

$$(1.36)^2 + 6(1.36) = 1.85 + 8.16 = 10.010$$

Since the value of x is to be rounded, the value of the expression in x is as close to 10 as is necessary.

So the value of x to satisfy the equation, (rounded to one decimal place) is 1.4

Algebraic Proportion

Direct proportion - If y is proportional to x, this can be expressed as:

$$y \propto x$$

$$y = kx$$

where k is 'the constant of proportionality'

A very useful equation can be obtained if we consider two sets of values of x and y.

$$x_1 \quad y_1 \qquad y_1 = kx_1 \qquad \text{(i}$$
$$x_2 \quad y_2 \qquad y_2 = kx_2 \qquad \text{(ii}$$

$$\frac{y_1}{y_2} = \frac{x_1}{x_2} \qquad \text{dividing (i by (ii}$$

There are 4 values here. Questions on direct proportion will give you 3 of these values and you will be required to work out the 4th.

Example #1 - A car travels 135 miles on 15 litres of petrol. How many miles will the car travel if it uses 25 litres?

$$y_1 = 135 \text{ miles}, \qquad x_1 = 15 \text{ litres}$$
$$y_2 = \ ? \qquad x_2 = 25 \text{ litres}$$

$$\frac{y_1}{y_2} = \frac{x_1}{x_2}$$

$$\frac{135}{y_2} = \frac{15}{25}$$

$$135 \times 25 = 15 y_2 \qquad \text{cross multiplying}$$

$$\frac{135 \times 25}{15} = y_2$$

$$y_2 = \frac{9 \times 25}{1} = 225$$

Answer: 225 miles

<u>Example #2</u> - The speed *v* of a rocket is directly proportional to the time *t* it travels. After 3 seconds its speed is 150 metres per second(m/s). How long after take-off will the speed reach 550m/s ?

$y_1 = 150$ m/s , $\quad x_1 = 3$ seconds

$y_2 = 550$ m/s , $\quad x_2 = ?$

$$\frac{y_1}{y_2} = \frac{x_1}{x_2}$$

$$\frac{150}{550} = \frac{3}{x_2}$$

$$150x_2 = 550 \times 3 \qquad \text{cross multiplying}$$

$$x_2 = \frac{550 \times 3}{150} = 11$$

<u>Answer: 11 seconds</u>

<u>Inverse proportion</u> - If y is inversely proportional to x, this can be expressed as:

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

where k is 'the constant of proportionality'

Another very useful equation can be obtained if we consider two sets of values of x and y.

$$x_1 \quad y_1 \qquad y_1 = \frac{k}{x_1} \qquad (i$$

$$x_2 \quad y_2 \qquad y_2 = \frac{k}{x_2} \qquad (ii$$

$$\frac{y_1}{y_2} = \frac{x_2}{x_1} \qquad \text{dividing (i by (ii}$$

As with the equation for direct proportion, there are 4 values here. Questions on inverse proportion will give you 3 of these values and you will be required to work out the 4th.

<u>Example</u> - It is assumed that the value of a second-hand car is inversely proportional to its mileage. A car of value £1200 has a mileage of 50,000 miles. What will its value be when it has travelled 80,000 miles?

$$y_1 = £1200\ , \qquad x_1 = 50{,}000 \text{ miles}$$

$$y_2 = \ ? \qquad x_2 = 80{,}000 \text{ miles}$$

$$\frac{y_1}{y_2} = \frac{x_2}{x_1}$$

$$\frac{1200}{y_2} = \frac{80000}{50000}$$

$$1200 \times 50000 = 80000 y_2 \qquad \text{cross multiplying}$$

$$\frac{1200 \times 50000}{80000} = y_2$$

$$y_2 = \frac{1200 \times 5}{8} = 150 \times 5 = 750$$

<u>Answer: £750</u>

Variation - This covers a number of proportionalities involving 'square', 'square root','cube root', 'cube', inverse or a combination of these.

The first thing you need to do is to write down the proportion in symbols and then as an equation. Here are some examples:

'a' is proportional to 'b' squared	$a \propto b^2$	$a = kb^2$
'c' is inversely proportional to 'd' cubed	$c \propto \frac{1}{d^3}$	$c = \frac{k}{d^3}$
'e' varies as the square root of 'f'	$e \propto \sqrt{f}$	$e = k\sqrt{f}$
'g' is proportional to 'h' cubed	$g \propto h^3$	$g = kh^3$
'i' varies as the inverse of 'j' squared	$i \propto \frac{1}{j^2}$	$i = \frac{k}{j^2}$

In questions on variation you are usually given a pair of x,y values and a proportionality. You are given one further value of x or y, and are required to calculate the missing value.

- find the 'constant of proportionality'(k) using the first 'xy' values and write down the proportionality as an equation
- put the new value of x or y in the equation and solve for the missing value

Example - If the value of y is proportional to the square of x, and x is 4 when y is 96, what is the value of y when x is 13?

$$x_1 = 4 \,, \quad y_1 = 96$$

$$y \propto x^2 \,, \quad y_1 = kx_1^{\,2}$$

$$96 = k(4^2)$$

$$96 = k(16)$$

$$\frac{96}{16} = k \,, \quad k = \frac{96}{16} = 6$$

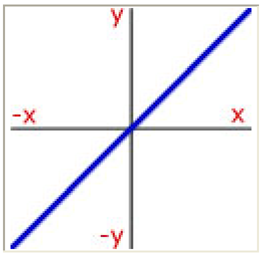
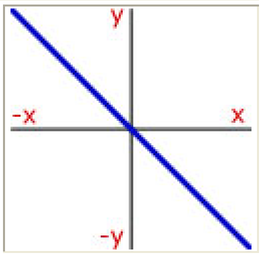
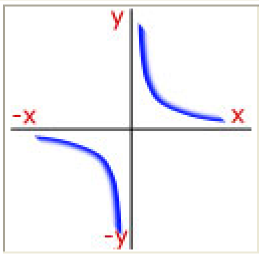
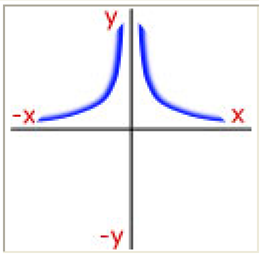
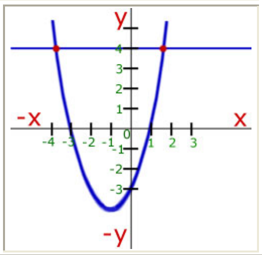
$$\underline{y = 6x^2}$$

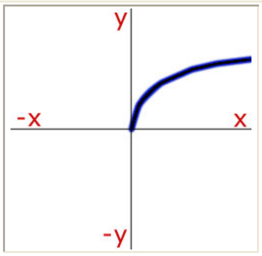
$$x_2 = 13 \quad y_2 = kx_2^{\,2}$$

$$y_2 = 6(13^2) \,, \quad y_2 = 6(169)$$

$$\underline{y_2 = 1014}$$

<u>Curve Sketching</u> - Try to remember the proportionality that matches the shape of the curve.

$y \propto x$	
$y \propto -x$	
$y \propto \frac{1}{x}$	
$y \propto \frac{1}{x^2}$	
$y \propto x^2$	

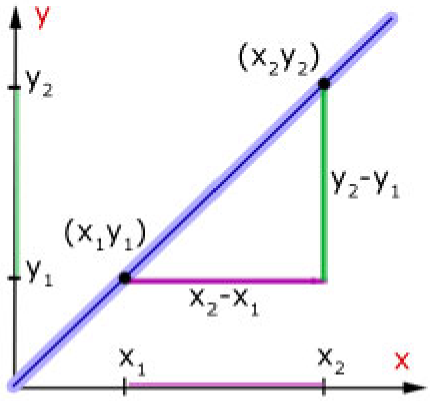
$y \propto \sqrt{x}$	

Linear Graphs

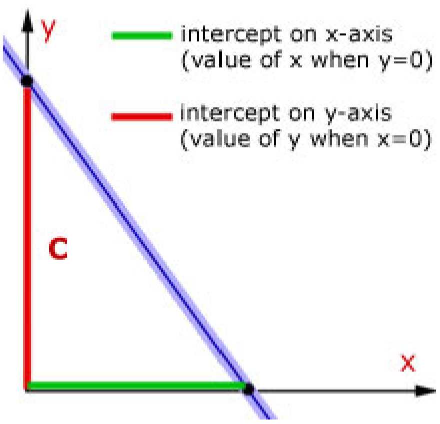
The equation of a straight line - This is given the form **y=mx+c**, where 'm' is the **gradient** of the graph and 'c' is the **intercept** on the y-axis(i.e. when x=0).

The **gradient(m)**of a line is the ratio of the 'y-step' to the 'x-step' from a consideration of two points on the line.

$$\text{gradient(m)} = \frac{\text{y-step}}{\text{x-step}} = \frac{y_2 - y_1}{x_2 - x_1}$$



The **intercept - c** is the value of y when x=0. The other intercept(the value of x when y=0) is not used directly.



Since the equation of a straight line is **y=mx+c**, just looking at the equation is enough to give the gradient and the intercept on the y-axis. **m** is the number infront of the x. **c** is the number after the x term.

Example #1 Complete the table:

equation	gradient	intercept on y-axis
y = x - 3	+1	-3
y = -3x + 4	-3	+4
y = 0.5x - 5	+0.5	-5
y + x = 1	-1	+1
x - y = 2	+1	-2

Example #2 Write down the equation of the straight line that goes through the points (2,1) and (5,7).

$$\text{gradient(m)} = \frac{\text{y-step}}{\text{x-step}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-1}{5-2} = \frac{6}{3} = 2$$

putting into the equation one set of xy values(2,1),

$$y = 2x + c$$
$$1 = 2(2) + c$$
$$1 = 4 + c\ ,\quad c = 1 - 4\ ,$$
$$c = -3$$

hence the equation is:

$$\underline{y = 2x - 3}$$

Parallel & perpendicular lines

All lines with the same gradient are parallel. However, remember in each case the intercept with the x and y axis will be different.

examples of parallel lines - note the value of 'c' in each case

$$y = 7x - 5$$
$$y = 7x + 2$$
$$y = 7x + 12$$
$$y = 7x - 3$$

When two straight lines intersect at 90 degrees to eachother(i.e. are perpendicular), the product of their gradients is **-1**

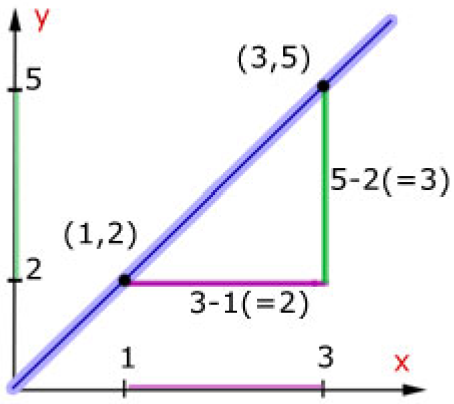
Example Complete the table of gradients of lines perpendicular to eachother

line #1 gradient	line #2 gradient
1	-1
-2	0.5
3	-0.333
-4	0.25
5	-0.2

The length of a line This is calculated using Pythagoras' Theorem.

The line between the two points is the hypotenuse of a right angled triangle. Draw horizontal and vertical lines from the points. Work out the lengths of the adjacent sides as you would to calculate gradient. Then use Pythagoras to calculate the hypotenuse.

Example Find the length of the line joining the points (1,2) and (3,5).



$$(line\ length)^2 = (side\,\#1)^2 + (side\,\#2)^2$$

$$(line\ length)^2 = (3)^2 + (2)^2$$
$$(line\ length)^2 = 9 + 4$$
$$(line\ length)^2 = 13$$
$$\underline{line\ length = \sqrt{13}}$$

The mid-point of a line - This is simply the average of the x-coordinate and the average of the y-coordinate.

Example - if we take the two points from the last example, (1,2) (3,5), the mid point is:

$$\left(\frac{1+3}{2}, \frac{2+5}{2}\right)$$

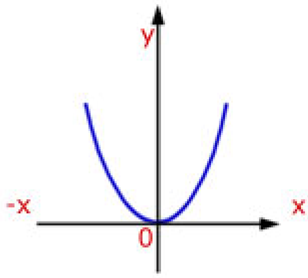
$$\left(\frac{4}{2}, \frac{7}{2}\right)$$

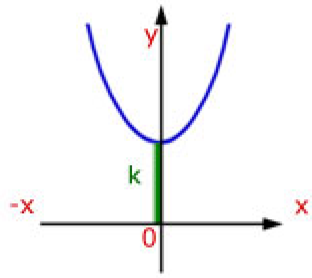
$$\underline{(2, 3.5)}$$

Graphs of Common Functions

The function....**y= f(x) + k**

$y = x^2$ $\qquad\qquad$ $y = x^2 + k$



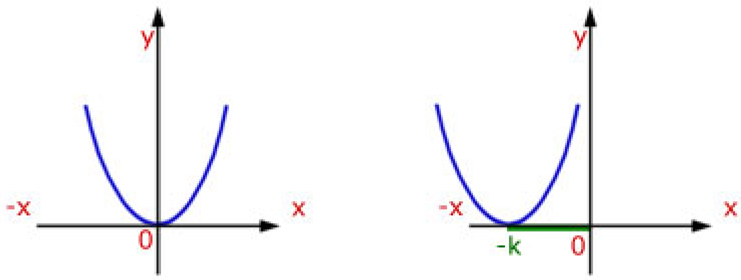


When x = 0, y = k . So the curve is moved(translated) by 'k' in the y-direction.

In vector terms the translation of the curve is $\begin{pmatrix} 0 \\ k \end{pmatrix}$

<u>The function</u>....**y= f(x + k)**

$$y = x^2 \qquad\qquad y = (x+k)^2$$



This is best understood with an example.

Let k be equal to some number, say 3. Adding 3 into the original equation, we have:

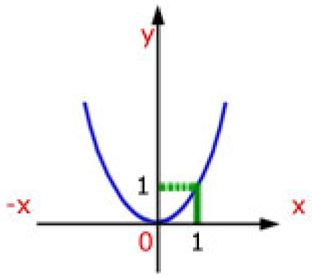
$$\begin{aligned} y &= f(x+3) \\ y &= (x+3)^2 \\ y &= (x+3)(x+3) \\ \text{when } y = 0,\ x &= -3 \end{aligned}$$

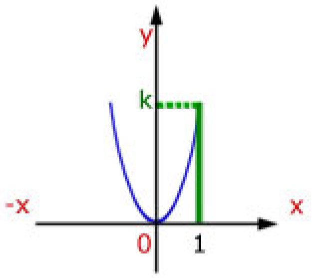
So the curve moves -3 to the left, to where y=0. That is -k to the left.

In vector terms the translation of the curve is $\begin{pmatrix} -k \\ 0 \end{pmatrix}$

<u>The function</u>....**y = kf(x)**

$y = x^2$ $\qquad\qquad$ $y = k(x)^2$





In our example, y increases by a factor of 'k' for every value of x.

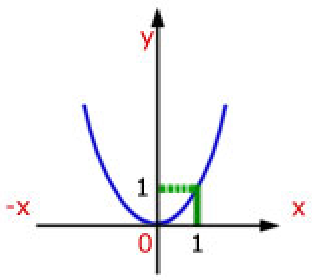
<u>Example</u> - let k=5

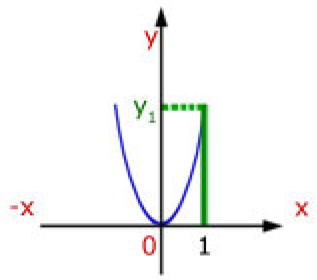
	$\underline{y = (x)^2}$	$\underline{y = 5(x)^2}$
$x = 1$	$y = (1)^2 = 1$	$y = 5(1)^2 = 5$
$x = 2$	$y = (2)^2 = 4$	$y = 5(2)^2 = 5(4) = 20$
$x = 3$	$y = (3)^2 = 9$	$y = 5(3)^2 = 5(9) = 45$

So for each value of x, the value of y is 5 times its previous value. The curve is stretched in the y-direction by a factor of 5. That is by a factor of k.

<u>The function</u>....**y= f(kx)**

$y = x^2$ $\qquad\qquad$ $y = (kx)^2$





In the above, when x=1, y=1. However, in the second function when x=1, y is a higher value. Look at the example below for x=1 and other values of x.

Remember, in this function the constant 'k' multiplies the x-value **inside** the function.

<u>Example</u> #1 - let k=4

	$y = (x)^2$	$y = (4x)^2$
$x = 1$	$y = (1)^2 = 1$	$y = (4 \times 1)^2 = 16$
$x = 2$	$y = (2)^2 = 4$	$y = (4 \times 2)^2 = 64$
$x = 3$	$y = (3)^2 = 9$	$y = (4 \times 3)^2 = 144$

You will notice that the y-value jumps by a factor of 16 for each increasing x-value. The y-value increases by a factor of 4 squared.

With more complicated functions the value of y for a given value of x, increases once more, narrowing the curve in the x-direction(or stretching in the y-direction).

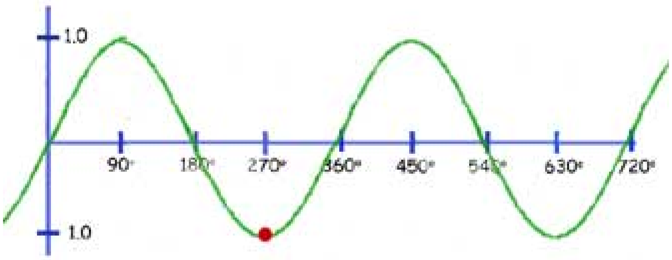
<u>Example</u> #2 a more complicated function with k=4

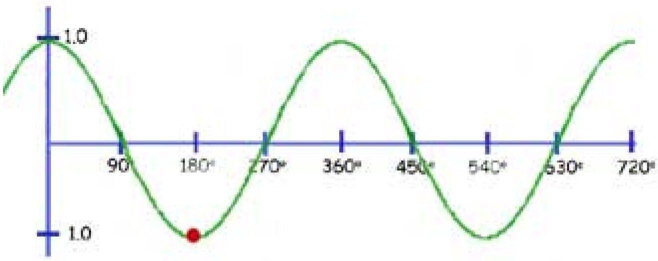
	$y = (x)^2 - 2(x) + 3$	$y = (4x)^2 - 2(4x) + 3$
	$y = x^2 - 2x + 3$	$y = 16x^2 - 8x + 3$
$x = 1$	$y = 1 - 2 + 3$	$y = 16(1) - 8(1) + 3$
$x = 1$	$y = 2$	$y = 16 - 8 + 3$
$x = 1$	$y = 2$	$y = 11$

<u>The function</u>....**y= sin(x+k)**

Here the graph is translated by the value of k, to the **left**
So when k=90 deg. The curve moves horizontally 90 deg. (looking at the red dot, from 270 deg. to 180 deg.)

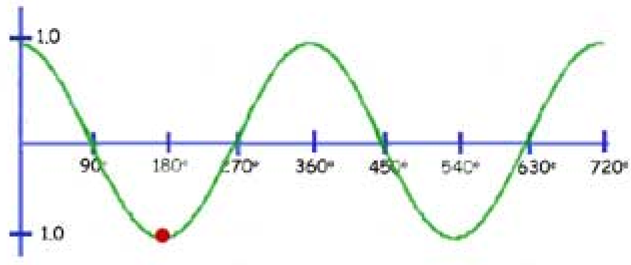
$$y = \sin(x)$$

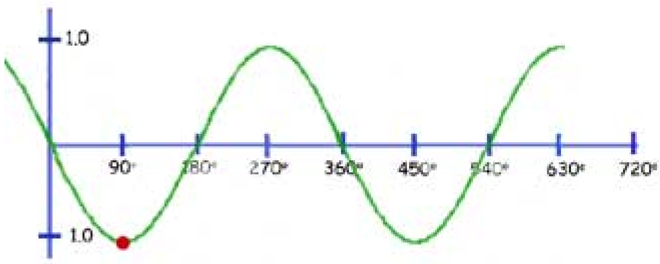




The function....**y= cos(x+k)**

This is exactly the same as for the sine function.
The graph is translated by the value of k, to the **left**
So when k=90 deg. The curve moves horizontally 90 deg. (looking at the red dot, from 180 deg. to 90 deg.)





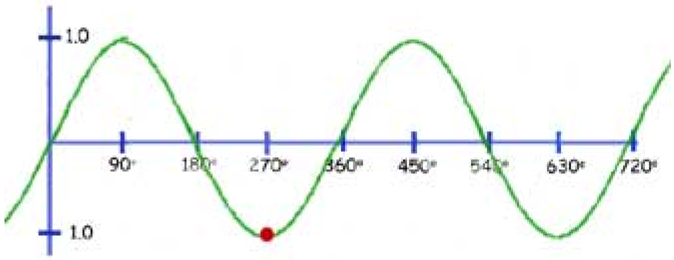
<u>The function</u>....**y= sin(kx)**

Here the graph is squeezed horizontally(concertinered) by a factor of k.

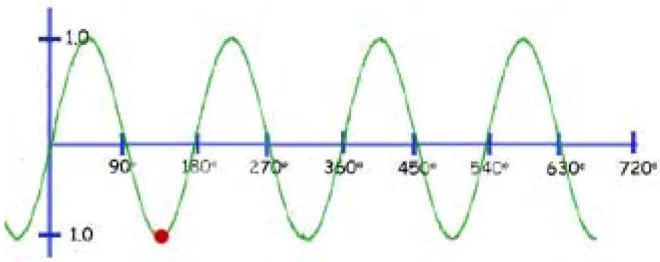
In our example below, k = 2. So one whole wavelength of 360 deg. is reduced to 180 deg.

Conversely you may think of any value of x being halved(red spot reading changes from 270 deg. to 135 deg)

$y = \sin(x)$



$y = \sin(2x)$



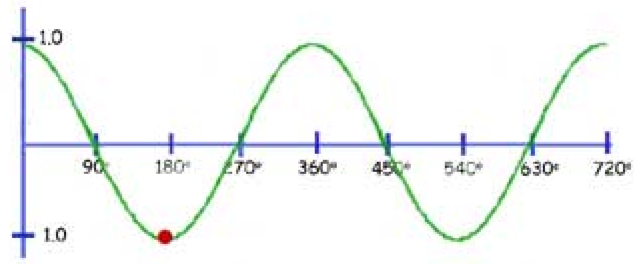
<u>The function</u>....**y= cos(kx)**

As with the previous function, the graph is squeezed horizontally(concertinered) by a factor of k.

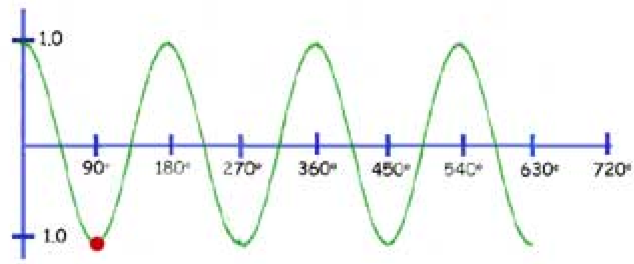
In our example below, k = 2. So one whole wavelength of 360 deg. is reduced to 180 deg.

Conversely you may think of any value of x being halved(red spot reading changes from 180 deg. to 90 deg).

$y = \cos(x)$



$y = \cos(2x)$



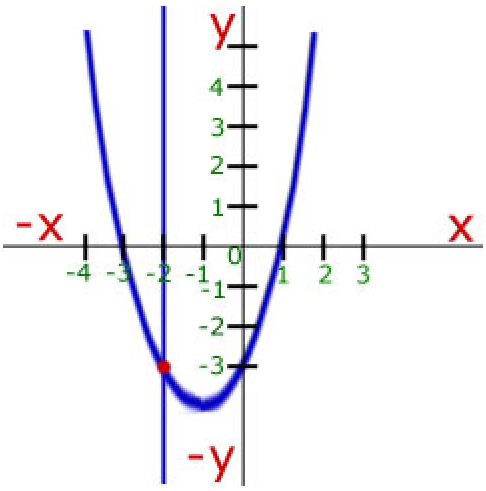
Graphical Solutions

A 'straight line intersecting straight line' is dealt with in the 'simultaneous equations' section.

Vertical line intersecting a quadratic curve

Example Find the point of intersection when the vertical at x=-2 meets the curve,

$$y = x^2 + 2x - 3$$



Substitute the value of x=-2 into the quadratic equation to find y.

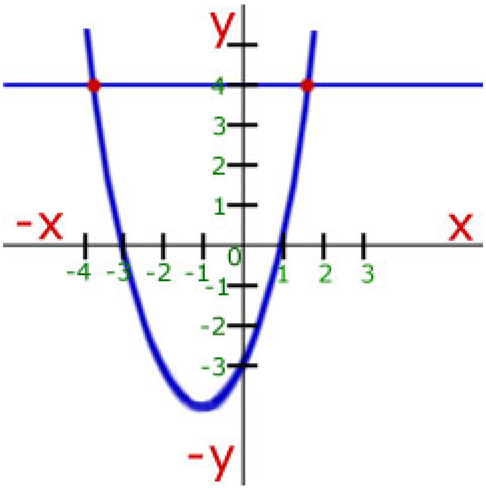
$$\begin{aligned} y &= x^2 + 2x - 3 \\ &= (-2)^2 + 2(-2) - 3 \\ &= 4 - 4 - 3 \\ &= -3 \end{aligned}$$

hence the point of intersection is (-2, -3)

Horizontal line intersecting a quadratic curve

Example - Find the two points of intersection when the horizontal at y=4 meets the curve,

$$y = x^2 + 2x - 3$$



To find the two points, put one equation equal to the other, rearrange putting zero on one side and find the roots.

$$x^2 + 2x - 3 = 4$$
$$x^2 + 2x - 7 = 0$$

The roots are complex, therefore we use the quadratic equation formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1, \quad b = 2, \quad c = -7$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-7)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4+28}}{2}$$

$$= \frac{-2 \pm \sqrt{32}}{2}$$

$$= -1 \pm \frac{\sqrt{32}}{2} = -1 \pm \frac{\sqrt{2 \times 16}}{2} = -1 \pm 4\frac{\sqrt{2}}{2} = -1 \pm 2\sqrt{2}$$

$$= -1 \pm 2\sqrt{2} = -1 \pm 2.828$$

$$\underline{x_1 = -1 + 2.828 = 1.828}$$

$$\underline{x_2 = -1 - 2.828 = -3.828}$$

check that x_1 and x_2 satisfy the quadratic and give $y = 4$

$$y_1 = x_1^2 + 2x_1 - 3 = (1.8)^2 + 2(1.8) - 3$$
$$= 3.2 + 3.6 - 3 = 3.8 \ \text{(approx.4)}$$

$$y_2 = x_2^2 + 2x_2 - 3 = (-3.8)^2 + 2(-3.8) - 3$$
$$= 14.4 - 7.6 - 3 = 3.8 \ \text{(approx.4)}$$

<u>The two points of intersection are (1.828, 4) and (-3.828, 4)</u>

N.B. the rounding of square roots makes the answers only approximate

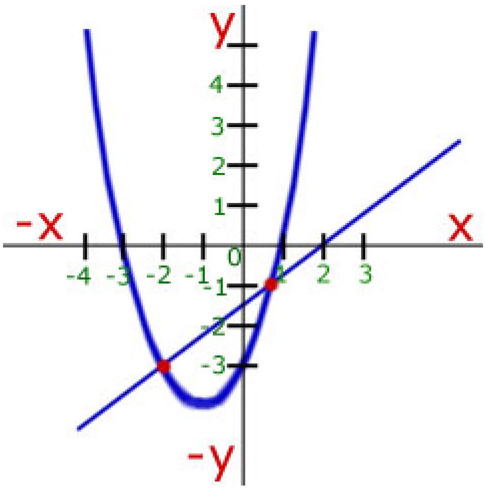
Angled straight line intersecting a quadratic curve

Example - Find the points of intersection when the straight line with equation,

$$y = \frac{3}{4}x - \frac{3}{2}$$

meets the curve,

$$y = x^2 + 2x - 3$$



As with the horizontal line intersection , the solution is to put one equation equal to the other, rearrange, put zero on one side and find the roots.

$$x^2 + 2x - 3 = \frac{3}{4}x - \frac{3}{2}$$

$$x^2 + \frac{5}{4}x - \frac{3}{2} = 0$$

$$a = 1, \quad b = 1.25, \quad c = -1.5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1.25 \pm \sqrt{(1.25)^2 - 4(1)(-1.5)}}{2(1)}$$

$$x = \frac{-1.25 \pm \sqrt{1.56 + 6}}{2} = \frac{-1.25 \pm \sqrt{7.76}}{2}$$

$$= \frac{-1.25 \pm 2.79}{2} = -0.63 \pm 1.39$$

$$\underline{x_1 = -0.63 + 1.39 = 0.76}$$

using the straight line equation $\underline{y_1 = 0.75(0.76) - 1.5 = -0.93}$

$$\underline{x_2 = -0.63 - 1.39 = -1.99}$$

also $\underline{y_2 = 0.75(-1.99) - 1.5 = 2.99}$

The two points of intersection are(0.76, -0.93) and (-1.99, 2.99)

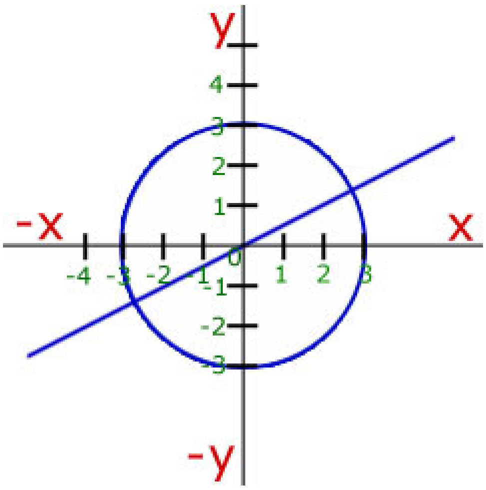
Straight line intersecting a circle

Example - Find the points of intersection when the straight line with equation,

$$y = \frac{x}{2}$$

meets the circle with equation,

$$x^2 + y^2 = 9$$



The solution is to take the y-value from the straight line equation and put it into the y-value of the circle equation. Then solve for x.

$$x^2 + \left(\frac{x}{2}\right)^2 = 9$$

$$x^2 + \frac{x^2}{4} = 9$$

$$4x^2 + x^2 = 36$$

$$5x^2 = 36\ , \quad x^2 = 7.2$$

$$\underline{x = \sqrt{7.2} = \pm 2.68}$$

$$\text{using} \quad y = \frac{x}{2}\ ,$$

$$\underline{y_1 = \frac{2.68}{2} = 1.34}$$

$$\underline{y_2 = \frac{-2.68}{2} = -1.34}$$

<u>The two points of intersection are(2.68, 1.34) and (-2.68, -1.34)</u>

Inequalities

Symbols

$x > y$	x greater than y
$x < y$	x less than y
$x \geq y$	x greater than or equal to y
$x \leq y$	x less than or equal to y

The rules of inequalities

These are the same as for equations i.e that whatever you do to one side of the equation(add/subtract, multiply/divide by quantities) you must do to the other.

However, their are **two** exceptions to these rules.

When you **multiply each side by a negative quantity**

'**<**' becomes '**>**', or '**>**' becomes '**<**' .

That is, the **inequality sign is reversed**.

Similarly, when you **divide each side by a negative quantity**

< becomes **>**, or **>** becomes**<** .

That is, the **inequality sign is reversed**.

Examples

$$-\frac{x}{2} < 6$$

multiplying each side by -2

$$\frac{-2}{-2}x > -12 \quad (\text{note } < \text{ to } >)$$

$$\underline{x > -12}$$

$$-5x > 4$$

dividing each side by -5

$$\frac{-5}{-5}x < \frac{4}{-5} \quad (\text{note } > \text{ to } <)$$

$$x < \frac{4}{-5}$$

$$\underline{x < -\frac{4}{5}}$$

Inequalities with one variable

Example #1 - Find all the integral values of x where,

$$6 \geq x > -5$$

The values of x lie equal to and less than 6 but greater than -5, but not equal to it.

The integral(whole numbers + or - or zero) values of x are therefore:

6, 5, 4, 3, 2, 1, 0, -1, -2, -3, -4

Example #2 - What is the range of values of x where,

$$x^2 \geq 144$$

Since the square root of 144 is +12 or -12(remember two negatives multiplied make a positive), x can have values between 12 and -12.

In other words the value of x is less than or equal to 12 and more than or equal to -12. This is written:

$$\underline{12 \geq x \geq -12}$$

Inequalities with two variables - Solution is by arranging the equation into the form

Ax + By = C

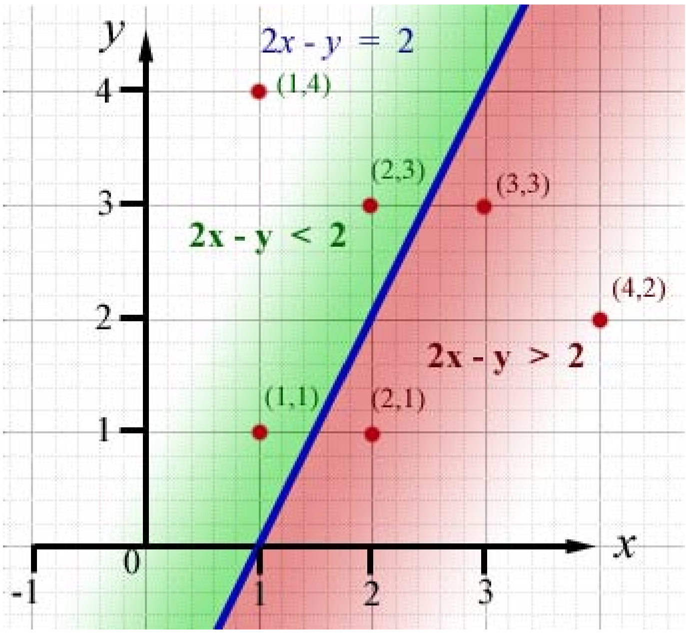
Then, above the line of the equation, **Ax + By < C**

and below the line, **Ax + By > C**

Consider the graph of -2x + y = -2

note - the first term **A** must be made positive by multiplying the whole equation by -1

The equation **-2x + y = -2** becomes **2x - y =2**



look at the points(red) and the value of **2x - y** for each. The table below summarises the result.

point(x,y)	2x - y	value	more than 2 ?	above / below curve
(1,1)	2(1)-(1)	1	no - less	above
(1,4)	2(1)-(4)	-2	no - less	above
(2,3)	2(2)-(3)	1	no - less	above
(3,3)	2(3)-(3)	3	yes - more	below
(2,1)	2(2)-(1)	3	yes - more	below
(4,2)	2(4)-(2)	6	yes - more	below

notes